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MODELLING AND DESIGN OF ROBOTIC SYSTEMS HAVING SPRING-DAMPER ACTUATORS

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Abstract: The role of inherent dynamics for the improvement of control strategies of robotic systems is studied. A mathematical formulation of the optimal control problem that is suitable for this investigation is proposed. In solving this problem closed-form expressions have been obtained for the optimal control strategies for n degrees-of-freedom robotic systems with passive (unpowered) drives and no restrictions upon their controlling stimuli, and with non-linear viscoelastic spring-damper actuators. The obtained results can be used in designing optimal spring-damper-like passive drives for robotic systems.

1. INTRODUCTION

Today our knowledge in mechanics, control engineering, electronics and computer science is actively integrated into a new interdisciplinary science - *mechatronics.* One of the primary goals of mechatronics is to gain as many advantages as possible of an optimal interaction between the mechanical, control, electronic and computer subsystems. This leads to a need to study many fundamental problems of mechanics and control engineering, which can give deeper insight into this interaction (Stadler, 1995; Bolton, 1999).

In this paper, the dynamics and optimal control problems are studied for robotic systems having passive actuators, e.g. springs, dampers, etc. The reasons for this study are as follows. Usually, the existing control systems for robots are designed under the assumption that a separate actuator governs each degree of freedom of the robot. This leads to complexity of the control system and to large energy consumption. It seems reasonable to attempt to simplify the control system of a robot by decreasing the number of actively controlled degrees of freedom. It can give great advantages to use different passive compliance elements like springs, dampers, brakes, etc. to control some degrees of freedom of a manipulator robot, e.g., during the performance of working tasks with periodic laws of motion (Akinfiev and Armada, 1998; Berbyuk and Kudyn, 1999).

The idea of the utilisation of inherent dynamics and passive compliance elements in the modelling of the motion and study of control problems of robotic and biorobotic systems was partially exploited by several investigators. For example, Formal'sky (1996) proposed an approach for the control of a bipedal locomotion mechanism based on the utilization of active short duration pulses at advantageous time intervals while allowing natural passive dynamics during the remainder of the walk. Garcia, et al. (1998) demonstrated that a simple, uncontrolled two-link model, vaguely resembling human leg, could walk down a shallow slope, powered only by gravity. This model is the simplest special case of the passive-dynamic models pioneered by McGeer (1990). Adolfsson, et al. (1998) also studied the inherent dynamics of a bipedal mechanism with particular emphasis on the existence of stable three-dimensional gait in the absence of external, actively regulated, control. Mennito and Buehler (1996) and Hardarson, et al. (1998) have demonstrated new compliant articulated robot legs, which were constructed as a prototype for an autonomous robot quadruped.

Previously Berbyuk, et al. (1998), have proposed an optimisation approach for the design of rotational spring-damper actuators providing the programmed goal-directed motion of a bipedal walking robot. The problem was formulated as an approximation procedure for the controlling torque acting at the joints of the robot during its optimal motion. The motion and the respective torques were determined by the solution of the optimal control problem for the dynamical system modelling the robot (Berbyuk, et al., 1999).

In this paper the problem of control and optimisation of robotic systems having both active (powered) and passive (unpowered) drives is considered. These robotic systems are called semi-passively actuated. The closed-form expressions are determined for the optimal control laws for two types of semi-passively actuated robotic systems: an n degrees-of-freedom robotic system having no restrictions on the controlling stimuli of the passive drives, and a robotic system with non-linear viscoelastic spring-damper actuators.

2. STATEMENT OF THE PROBLEM

Consider a manipulator robot with n degrees-of-freedom. Using the Lagrangian approach the controlled motion of the robot can be described by the following equations:

$$A(q)\ddot{q} + B(q,\dot{q}) = C(q)u(t) \qquad (1)$$

Here $q = (q_1, q_2, ..., q_n)$ is a vector of the generelized coordinates, $u = (u_1, u_2, ..., u_m)$ is a vector of the controlling stimuli (forces, torques) generated by the active (powered) drives of the robotic system, $A(q)$, $B(q,\dot{q})$, $C(q)$ are given matrices.

Assume that several springs, dampers and some other elements have been incorporated into the structure of the manipulator robot. These springs and dampers (passive drives) exert additional forces and/or torques due to the action of the external controlling forces upon the system and also due to motion of its respective parts. Assuming that the masses of the passive drives are negligibly small in comparison with the masses of other links of the system the equations of motion of the semi-passively actuated manipulator robot can be written as follows:

$$A(q)\ddot{q} + B(q,\dot{q}) = C(q)u(t) + D(q)w(q,\dot{q}) \qquad (2)$$

Here $w = (w_1, w_2, ..., w_r)$ is a vector of the controlling stimuli of the passive drives,

$D(q)$ is a matrix which is determined by the structure of the passive drives.

Usually some constraints and restrictions are imposed on the phase coordinates $q(t)$, $\dot{q}(t)$, the controlling stimuli of the passive drives $w(q,\dot{q})$, and the external control laws $u(t)$ of the system. These restrictions can be written in the following way:

$$\{q(t), \dot{q}(t)\} \in Q, \qquad t \in [0,T] \tag{3}$$

$$w(q,\dot{q}) \in W, \qquad t \in [0,T] \tag{4}$$

$$u(t) \in U, \qquad t \in [0,T] \tag{5}$$

In formulas (3) - (5), Q and U are given domains in the phase and control spaces of the system, respectively; W is a set of addmissible controlling stimuli determined by the structure of the passive drives; T is the duration of the controlled motion of the robotic system, e.g. the duration of a pick-and-place operation.

The differential equation (2) together with the restrictions (3)-(5) is called the mathematical model of the semi-passively actuated robotic system. This model can be used for many applications, e.g. for computer simulations of the motion of closed-loop chain manipulator robots with passive drives (Lidberg and Berbyuk, 2000), for the study of control strategies for the stable motion of bipedal locomotion systems with compliance elements at the joints (Berbyuk, 1996), etc.

Assume that there exists a non-empty set of vector-functions

$$\{q(t), u(t), w(q,\dot{q}), \quad t \in [0,T]\}$$

which satisfy the equation (2) and the constraints (3)-(5). The following optimal control problem can be formulated.

Problem A. Given a robotic system the controlled motion of which is described by equation (1). It is required to determine the vector-function of passive drives $w_*(q,\dot{q})$, the motion of the system $q_*(t)$ and the external controlling stimuli $u_*(t, q_*, w_*)$ which alltogether satisfy the equation (2), the restrictions (3)-(5), and which minimize the given objective functional $\Phi[u(t)]$.

As a result of the solution of *Problem A* the optimal structure of the robotic system having both powered and unpowered drives is designed. The external controlling stimuli for the system is also found which minimize the given objective functional.

One of the primary goals for the incorporation of passive drives into the structure of robotic systems is an improvement of their control processes. It means that the validity of the following inequality is expected:

$$\Phi[u_*(t, q_*, w_*)] < \Phi[u_{0*}(t, q_{0*})]$$

where $q_{0*}(t)$, $u_{0*}(t)$ are the optimal motion and the controlling stimuli of the robotic system (1) obtained under the restrictions (3), (5). In this sense the solution of *Problem A* could help to estimate the limiting possibility of improvement of the external control strategies for robotic systems due to incorporation of different passive drives determined by the constraints (4) into their structure.

In the general case to solve *Problem A* for robotic systems with many degrees-of-freedom powerful numerical algorithms are needed. Futhermore, during the calculation of optimal control laws for a robotic system it is necessary to design at the same time the optimal structure of the passive drives taking into account the restriction (4). This can significantly increase the complexity of the computation.

Problem A was solved previously by Berbyuk (1996, 1997, 1999) for a 2D model of a bipedal locomotion system having springs and pneumatic or hydraulic actuators at their joints. The obtained results can be used for design of optimal lower limb prostheses and locomotion machines having compliance elements at their legs.

Closed-form solutions of *Problem A* obtained for some practically important cases are presented in the next paragraph of this paper.

3. OPTIMAL PASSIVE DRIVES FOR GIVEN MOTION OF A ROBOTIC SYSTEM

Below it is assumed that every degree-of-freedom of the robotic system has one external drive and also one passive drive. It means that the parameters *n, m* and *r* which correspond to the dimensions of the vectors *q(t), u(t)* and $w(q,\dot{q})$, respectively, are supposed to be equal.

Let an admissible motion $q_0 = q_0(t)$ of the robotic system (1) be given. To execute this motion by the robotic system it is necessary to apply the following external controlling stimuli:

$$u_0(t) = C^{-1}(q_0)(A(q_0)\ddot{q}_0 + B(q_0,\dot{q}_0)) \quad (6)$$

For the same motion $q_0 = q_0(t)$ the external controlling stimuli of the robotic system with passive drives can be calculated using the equation (2) and the formula (6). These controlling stimuli can be represented as follows:

$$u_w(t,q_0) = u_0(t) - C^{-1}(q_0)D(q_0)w(q_0,\dot{q}_0) \quad (7)$$

where $u_0(t)$ is determined by the formula (6).

Assume that there are no restrictions on the passive drives of the robotic system. Then from the formula (7) follows that the passive drives exerting the controlling stimuli of the kind

$$w_* = D^{-1}(q_0)C(q_0)u_0(t), \quad t \in [0,T] \quad (8)$$

are optimal ones. This is in the sense that the given motion $q_0 = q_0(t)$ can be fully executed only by means of the controlling stimuli of the passive drives.

Consider a manipulator robot having *n* degrees-of-freedom. Let the equation of its controlled motion be as follows:

$$A(q)\ddot{q} + B(q,\dot{q}) = u(t), \qquad t \in [0,T] \quad (9)$$

The external controlling stimuli are studied that transfer the robot from the initial phase state

$$q(0) = q^0, \qquad \dot{q}(0) = \dot{q}^0 \quad (10)$$

to the final phase state

$$q(T) = q^T, \qquad \dot{q}(T) = 0 \quad (11)$$

Here $(q^0,\dot{q}^0)$, $(q^T,0)$ are given points in the phase space of the system, and *T* is the duration of the control process.

At the same time, assume that the robot has additional passive drives, namely non-linear visco-elastic spring-damper actuators in its structure. The mathematical model of the semi-passively actuated manipulator robot can be written as follows:

$$A(q)\ddot{q} + B(q,\dot{q}) = u(t) + w(q,\dot{q}) \quad (12)$$

$$w(q,\dot{q}) + kf(q,\dot{q}) = 0, \qquad t \in [0,T] \quad (13)$$

where the function $f(q,\dot{q})$ determines the inherent dynamics of the passive drives under the restriction (4) and *k* is a damper coefficient.

To estimate the quality of the control processes the following objective functional is exploied

$$\Phi[u(t)] = \int_0^T \|u(t)\|^2 dt \tag{14}$$

where $\|u(t)\| = (u_1^2(t) + ... + u_n^2(t))^{1/2}$.

In many cases this functional can be used to estimate the energy consumption needed for the controlled motion of the mechanical systems driven by the electromotors (Athans et al., 1963; Krasovskii, 1968).

Let $\{q_0(t),\ u_0(t),\ t \in [0, T]\}$ be any pair of functions that satisfy equation (9) and boundary conditions (10), (11). Then, as follows from equation (9), the functional (14) will be equal to

$$\Phi[u_0(t)] = \int_o^T \|u_0(t)\|^2 dt \tag{15}$$

where

$$u_0(t) = A(q_0)\ddot{q}_0(t) + B(q_0, \dot{q}_0) \tag{16}$$

It is assumed that the motion $\{q_0(t),\ t \in [0, T]\}$ can also be realised by the considered semi-passively actuated robot. Using equations (12), (13), the external controlling stimuli are written as follows

$$u_{w0}(t) = u_0(t) + kf(q_0, \dot{q}_0) \tag{17}$$

where $u_0(t)$ is determined by the formula (16).

For the control law (17) the objective functional will be equal to

$$\Phi[u_{w0}(t)] = \int_0^T \|u_0(t) + kf[q_0(t), \dot{q}_0(t)]\|^2 dt$$

This formula can also be written as follows:

$$\Phi[u_{w0}(t)] = \Phi[u_0(t)] + ak^2 + 2bk \tag{18}$$

where

$$a = \int_0^T \|f(q_0, \dot{q}_0)\|^2 dt$$

$$b = \int_0^T \langle u_0(t), f(q_0, \dot{q}_0) \rangle dt \tag{19}$$

$$\langle u_0(t), f(q_0, \dot{q}_0) \rangle = (u_{01} f_1 + ... + u_{0n} f_n)$$

It can be shown that the function (18) has a global minimum with respect to the damper coefficient k. The value of this minimum is equal to

$$\Phi_{\min} = \Phi[u_0(t)] - b^2 / a \tag{20}$$

for the following optimal damper coefficient

$$k_* = -b / a \tag{21}$$

In the formulas (20) and (21) the parameters a and b are determined by the expressions (19).

The above makes it possible to conclude that for a manipulator robot with n degrees-of-freedom and for any motion $\{q_0(t),\ t \in [0, T]\}$ satisfying the two-point boundary conditions (10), (11) the energy-optimal non-linear visco-elastic spring-damper actuators are determined by the formulas (13), (19), (21).

As follows from (19) and (20) the decrease in energy consumption due to the incorporation of the optimal spring-damper actuators into the robot structure is equal to

$$\Phi[u_0(t)] - \Phi_{\min} = \left\{ \int_0^T \langle u_0(t), f(q_0, \dot{q}_0) \rangle dt \right\}^2 / \int_0^T \|f(q_0, \dot{q}_0)\|^2 dt$$

This value depends only on the given motion $\{q_0(t),\ t \in [0, T]\}$ and the function $f(q, \dot{q})$ determining the inherent dynamics of the passive drives.

Usually some restrictions are imposed on the external controlling stimuli $u(t)$. In this case the function $f(q,\dot{q})$ can not be choosen arbitrarily. Indeed, let us assume that the external controlling stimuli $u(t)$ are restricted by the constraint

$$\|u(t)\| \le u_{\max}, \qquad t \in [0,T] \tag{22}$$

with given positive number $u_{\max}$. Then as follows from the formulas (17) and (21) the function $f(q,\dot{q})$ must satisfy not only the restriction (4) but also the inequality

$$\|u_0(t) - bf(q_0,\dot{q}_0)/a\| \le u_{\max}, \quad t \in [0,T]$$

where $u_0(t)$, a and b are determined by the expressions (16) and (19).

Example. Consider a system the controlled motion of which is described by the equation

$$\ddot{x} = u(t) \tag{23}$$

and the restriction

$$|u(t)| \le u_0, \qquad t \in [0,T] \tag{24}$$

Let the given motion of the system be determined by the formula

$$x(t) = x_0 + V_0 t - u_0 t^2/2, \qquad t \in [0,T] \tag{25}$$

where x_0, V_0, u_0 are given parameters, and $V_0 = u_0 T$.

For the motion (25) the objective function (14) is equal to

$$\Phi[u(t)] = u_0^2 T \tag{26}$$

Consider the same system but with an additional visco-elastic damper-like passive drive. The equations of motion of the obtained semi-passively actuated system are as follows

$$\ddot{x} = u(t) + w(\dot{x}), \qquad w + k\dot{x}(t) = 0 \tag{27}$$

where k is the damping coefficient of the passive drive.

The control stimuly needed to execute the motion (25) by this system can be written by the formula

$$u_w(t) = k(V_0 - u_0 t) - u_0 \tag{28}$$

and the respective value of objective function (14) is equal to

$$\Phi_w = \int_o^T [k(V_0 - u_0 t) - u_0]^2\, dt \tag{29}$$

The function $\Phi_w(k)$ has a global minimum in parameter k. This minimum is equal to

$$\Phi_{w*} = u_0^2 T - 3u_0 V_0/4 \tag{30}$$

for the damping parameter value:

$$k_* = 3u_0/(2V_0) \tag{31}$$

Using the formulas (28) and (31) the optimal external control stimuly for the semi-passively actuated system which satisfy the restriction (24) can be written in the way

$$u_{w*}(t) = 3u_0(V_0 - u_0 t)/(2V_0) - u_0 \tag{32}$$

As follows from (26) and (30) the decrease in energy consumption due to the incorporation of the optimal damper-like actuator into the system's structure is equal to

$$\Phi[u(t)] - \Phi_{w*} = 3u_0 V_0/4$$

and this value depends only on the parameters of the given motion (25).

4. DISCUSSION AND CONCLUSION

In this paper the fundamental problem of optimal interaction between the controlling stimuli generated by active (powered) drives and passive (unpowered) drives of robotic systems has been studied. This problem is one among several general problems about the role of inherent dynamics in control of mechatronic systems. For instance, it is very important to know how much the mechatronic system should be governed by the external drives and how much by the system's inherent dynamics.

A mathematical statement of the problem (*Problem A*) is proposed, which is suitable for studying the above problems for semi-passively actuated n degrees-of-freedom robotic systems. This problem is formulated as an optimal control problem for an n degrees-of-freedom mechanical system modelling a semi-passively actuated robot. The interaction between the external controlling stimuli acting upon the robotic system and the control forces and/or torques exerted by its passive drives is considered. This is made by introducing the additional constraints imposed both on the phase coordinates and the controlling stimuli of the passive drives. The specification of these constraints (formula (4)) depends on the type and structure of the passive drives. Usually the constraint (4) is some additional differential equations describing the inherent dynamics of the passive drives incorporated into the structure of the robotic system.

In the present paper the closed-form solution of *Problem A* has been obtained for two cases. First, for an arbitrary robotic system having n degrees-of-freedom without any restriction on the controlling stimuli for its passive drives. Second, for an arbitrary robotic system with n degrees-of-freedom having non-linear visco-elastic spring-damper actuators. In both cases the motion of the robotic system is assumed to be specified in advance.

The analysis of the obtained results shows that in several cases the incorporation of passive drives into the structure of a robotic system can decrease the energy consumption needed for the given motion of the system.

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REFERENCES

Adolfsson, J., H. Dankowicz and A. Nordmark (1998). 3-d stable gait in passive bipedal mechanisms. In: Proceedings of the EUROMECH Colloquium *"Biology and Technology of Walking"*, Munich, Germany.

Akinfiev, T. and M. Armada (1998). Resonance and quasiresonance drives for start-stop regime. In: *Mechatronics 98,* (J. Adolfson and J. Karlsén, (Ed)), 91-96, Elsevier, The Netherlands.

Athans, M., P.L. Falb and R.T. Lacoss (1963). Time-fuel-and energy-optimal control of nonlinear norminvariant systems. *IEEE, Trans. Automat. Contr.,* 8, 3, 196-202.

Berbyuk, V. (1996). Multibody system modelling and optimization problems of lower limb prostheses. In: *IUTAM Symposium on Optimization of Mechanical Systems,* (D. Bestle and W. Schiehlen, (Ed)), 25-32, Kluwer Academic Publisher, The Netherlands.

Berbyuk, V. (1997). Dynamics and optimal control of biotechnical system "Man-Prosthesis". In: *IUTAM Symposium on Interaction between Dynamics and Control in Advanced Mechanical Systems,* (D. van Campen, (Ed)), 35-43, Kluwer Academic Publisher, The Netherlands.

Berbyuk, V. (1999). Dynamic simulation of human gait and design problems of lower limb prostheses. In: *Proceedings of the XIIth Biomechanics Seminar,* (Christian Högfors, (Ed)), 1-20, Göteborg.

Berbyuk, V., A. Boström, B. Lytwyn and B. Peterson (1999). Optimization of control laws of bipedal locomotion systems. In: *Advances in Computational Multibody Dynamics,* (Jorge A.C. Ambrósio and Werner O. Schiehlen, (Ed)), 713-728, Instituto Superior Técnico, Lisbon.

Berbyuk, V. and M. Kudyn (1999). The mathematical modelling of the sub-optimal motion of the semi-passively controlled mechanical systems. *Mathematical Methods and Physicomechanical Fields,* 42, 127-134.

Berbyuk, V., B. Peterson and N. Nishchenko (1998). Linear viscoelastic actuator-based control system of a bipedal walking robot. In: *Mechatronics 98,* (J. Adolfson and J. Karlsén, (Ed)), 379-384, Elsevier, The Netherlands.

Bolton, W. (1999). *Mechatronics*, 2nd Edition, Addison Wesley Longman Publishing, New York.

Formal'sky, A. (1996). Ballistic locomotion of a biped. Design and control of two biped machines. In: *Advanced School on Modeling and Simulation of Human and Walking Robots Locomotion,* (A. Morecki and K.J. Waldron, (Ed)), 191-229, CISM, Udine, Itali.

Garcia, M., A. Chatterjee, A. Ruina and M. Coleman (1998). The simplest walking model: Stability, complexity, and scaling. *ASME, J. of Biomechanical Engineering,* 120, 281-288.

Hardarson, F., T. Benjelloun, T. Wadden and J. Wikander (1998). Biologically inspired design and construction of a compliant robot leg for dynamic walking. In: *Mechatronics 98,* (J. Adolfson and J. Karlsén, (Ed)), 367-372, Elsevier, The Netherlands.

Krasovskii, N.N. (1968). *Theory of Control of Motion*. Nauka, Moscow.

Lidberg, M. and V. Berbyuk (2000). Modeling of controlled motion of semi-passively actuated SCARA-like robot. In: *Mechatronics 2000, 6-8 September 2000*, Atlanta, USA, Elsevier.

McGeer, T. (1990). Passive dynamic walking. *International Journal of Robotics Research,* 9, 62-82.

Mennito, G. and M. Buehler (1996). CARL: A compliant articulated leg for dynamic locomotion, *J. Robotics and Autonomous Systems,* 18, 337-344.

Stadler, W. (1995). *Analytical Robotics and Mechatronics*. McGraw –Hill, New York.